# Optimization of Integrated Departures and Arrivals Under Uncertainty

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In terminal airspace, integrating arrivals and departures with shared waypoints provides the potential of improving operational efficiency by allowing direct routes when possible. Incorporating stochastic evaluation as a post-analysis process of deterministic optimization is one way to learn the impact of uncertainty and to avoid unexpected outcomes. This work presents a way to take uncertainty into consideration during the optimization. The impact of uncertainty was incorporated into cost evaluations when searching for the optimal solutions. The controller intervention count was computed using a heuristic model and served as another cost besides total delay. Costs under uncertainty were evaluated using Monte Carlo simulations. The Pareto fronts that contain a set of solutions were identified and the trade-off between delays and controller intervention count was shown. Solutions that shared similar delays but had different intervention counts were investigated. The results showed that optimization under uncertainty can identify compromise solutions and help decision-makers reduce controller intervention while achieving low delays.

## I. Introduction

In the National Airspace System (NAS) terminal airspace is often busy and complicated because hundreds of flights must fly through a limited airspace in a short time period and most flights in terminal areas are climbing and descending with varied speeds. Situations can become severe when there are several neighboring busy airports. Bottlenecks can be formed easily and impair the efficiency of air traffic operations. Therefore, improving the operation efficiency in terminal airspace is critical for building an efficient air traffic system.

In past years, to improve the operational efficiency in terminal airspace, some researchers attempted to solve arrival scheduling problems.<sup>1–6</sup> Still others<sup>7–9</sup> addressed airport surface management problems, which are correlated to the efficiency of terminal airspace operations. When different arrival and/or departure flows share the same resources such as runways, waypoints and/or route segments, inefficient operations emerge because of the constraints of shared resources. Such interactions can happen among departures, arrivals, or between departures and arrivals. Recent studies<sup>10–12</sup> showed that optimized integrated arrivals and/or departures in major airports or metroplex areas have promise for improving operation efficiency. However, the benefits from optimal schedules calculated under deterministic scenarios are usually sensitive to flight time uncertainties, which can be caused by many sources, such as inaccurate wind prediction, error in aircraft dynamics, or human factors. Therefore, the impact of uncertainty must be taken into account when evaluating a schedule's benefits. Incorporating stochastic evaluation as a post-analysis process of deterministic optimization as in previous work<sup>13</sup> is one passive way to learn the impact of uncertainty and to avoid unexpected results. Optimizing integrated arrivals and departures under uncertainty is a proactive method that directly takes uncertainty into account. A set of compromise solutions can be identified and offered directly to decision makers to avoid unexpected effects from uncertainty.

This work presents an optimization method of integrating arrivals and departures under uncertainty, in which the impact of uncertainty was measured directly in the optimization. The optimization is multi-

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objective including total delay and controller intervention count. Both costs were evaluated using Monte-Carlo simulations. To enable the time-consuming optimization, the number of Monte-Carlo simulations were decreased. The impact of simplified Monte-Carlo simulation was examined. The Pareto fronts that contain sets of solutions were presented to show the trade-off between delays and interventions. Solutions with similar delays but different controller intervention count were investigated further.

In the paper, Section II revisits the problem modelled in previous work. Section II also presents the multiple objective optimization method that involves Monte Carlo simulations. Section III provides the results and analysis. Section IV provides conclusions for this work.

# II. Background and Problem

The interactions between arrivals and departures in Los Angeles terminal airspace were presented in previous work<sup>12</sup> for studying optimal integrated operations in deterministic circumstances. The uncertainty analysis of the deterministic solutions has been conducted in other work<sup>13</sup> as determined from post-processing. This work is focused on the same problem of integrating arrivals and departures in Los Angeles.



Figure 1. Interactions between SADDE arrivals and CASTA departures

According to the Standard Terminal Arrival Routes (STARs) and the Standard Instrument Departures (SIDs) of Los Angeles terminal airspace, the arrivals from the FIM fix would follow procedure SADDE6 (FIM-SYMON-SADDE-SMO) and the departures to the North need to follow procedure CASTA2 (RWY-NAANC-GHART-SILEX) (see Fig. 1). The arrivals are requested to maintain their flight altitudes above 12,000 feet at fix GHART and the departures have to keep theirs at or below 9,000 feet at the same fix to procedurally avoid potential conflicts between arrivals and departures. If there were no interactions, departures to the north and arrivals from FIM would have flown direct routes. As shown in the Fig. 1, the direct routes would be RWY-WPT2-WPT1 and FIM-WPT1-SMO for departures and arrivals, respectively, where WPT1 and WPT2 are made-up fix names for simplicity. Compared to these direct routes, besides flying non-preferred altitudes, individual arrival and departure flights following current procedures will waste approximately 60 and 120 seconds, respectively.

Table 1 shows a representative schedule of 14 flights, which covers half an hour traffic in actual operations. Two flows are included: 6 departures to the North from Runway 24L (RWY) and 8 arrivals from FIM. This schedule used the traffic between 9:00am and 9:30am (local time) on December 4, 2012 as reference. The initial times shown in the table are relative times to simulation start time. The "Order" of each flight is sorted based on intial times.

Table 1. Scheduled initial times

Order	FIM (sec)	RWY (sec)
1	39	68
2	446	165
3	728	363
4	1106	529
5	1332	1613
6	1475	1830
7	1613	NA
8	1770	NA

#### III. Method

In this study, the model is constructed similarly to previous works.<sup>12,13</sup> Unlike pervious work, the optimization is formulated as a multiple objective optimization. The first objective is to minimize delay and the second objective is to minimize controller intervention, which is believed to associate with controller workload increase. Both objectives were evaluated using Monte Carlo simulations.

#### A. Model

Three separation methods were compared in previous work:<sup>12</sup> spatial, temporal, and hybrid. Spatial separation uses the same strategy as in SIDs and STARs to spatially separate interacting departure and/or arrival flows. Temporal separation utilizes the direct routes with conflicts resolved merely with temporal control. Hybrid separation applies both temporal and spatial separations. This work was focused on hybrid separation.

In the formulation of hybrid separation, four design variables were defined for each FIM arrival i:  $d1_{(FIM,i)}$  is the delay before or at FIM;  $r_{(FIM,i)}$  is for route option, where 0 denotes the direct route and 1 denotes indirect route;  $v_{(FIM,i)}$  is the aircraft speed between FIM and WPT1 for the direct route or the speed between FIM and WPT2 if the indirect route is chosen;  $d2_{(FIM,i)}$  is the delay between WPT1/WPT2 and SUTIE to ensure separation at SUTIE. For a departure flight j, three decision variables were defined:  $d_{(DEP,j)}$  is the delay before departure;  $r_{(DEP,j)}$  is the route option, where 0 denotes the direct route and 1 denotes the indirect route.  $v_{(DEP,j)}$  is the speed from departure to WPT1.

Separation requirements were applied as hard constraints at fixes that could have potential violations, such as FIM, RWY, WPT1, WPT2, and SUTIE. Separation requirements were 3 nmi at the RWY and 4 nmi elsewhere. Additionally, an uncertainty buffer  $\delta$  was added in the separation constraints. For example, Eqns. 1, 2, and 3 show separation constraints for crossing flights between FIM arrivals and departures at WPT1, where  $t_{FIM(WPT1,i)}$  is the arrival time of FIM arrivals at fix WPT1 and  $t_{DEP(WPT1,j)}$  is the arrival time of DEP departures at fix WPT1. Eqn. 1 showed that if both arrival flight i and departure flight j took direct routes the separation must be satisfied. The separation requirement is 4.0 nmi in distance and in time scale the separation depends on the speeds of both flights.

$$(1 - r_{(FIM,i)}) \cdot (1 - r_{(DEP,j)}) \cdot [t_{FIM(WPT1,i)} - t_{DEP(WPT1,j)}] - \frac{4.0 \times 3600.0}{V \cdot sin\alpha} - \delta > 0 \tag{1}$$

$$V = \begin{cases} v_{(FIM,i)}, & if \ t_{FIM(WPT1,i)} < t_{DEP(WPT1,j)} \\ v_{(DEP,j)}, & otherwise \end{cases}$$
 (2)

$$\alpha = \begin{cases} atan\left[\frac{v_{(DEP,j)}}{v_{(FIM,i)}}\right], & if \ t_{FIM(WPT1,i)} < t_{DEP(WPT1,j)} \\ atan\left[\frac{v_{(PIM,i)}}{v_{(DEP,j)}}\right], & otherwise \end{cases}$$
(3)

### B. Objectives

The optimization in previous work was deterministic with a single objective. The objective was to minimize the total delay of all departures and arrivals. In this work, the problem is formulated as a multiple objective optimization, where controller intervention is considered as another objective. The objectives are shown in Eqn. 4, where N is the controller intervention count.

$$\begin{cases}
J_1 = \sum_{i,j,k} t_{FIM(SUTIE,i)} + t_{DEP(WPT1,j)} \\
J_2 = \sum_{i,j,k} N
\end{cases}$$
(4)

Delay cost  $J_1$  is equivalent to the sum of flight's exit times. For arrivals and departures, they are the times when flights reach SUTIE and WPT1, respectively. The difference from previous study<sup>12</sup> is that the delay in this work is evaluated stochastically using Monte Carlo simulations instead of evaluating deterministically. In Monte Carlo simulations, errors that follow normal distributions are introduced in flights' entry times. For a FIM arrival, the perturbation is introduced into the estimated arrival time at FIM. For a westbound arrival, the error is added to the estimated arrival time at SUTIE. Whereas for a departure, such error is imposed on the estimated takeoff time from the runway.

As in previous works, <sup>12,13</sup> a heuristic model was built to mimic controller intervention behaviors. When stochastic errors are added in flights' entry times, the heuristic model resolves potential conflicts by imposing extra delays to corresponding aircraft, while keeping the same route options as in the given solution. The extra delays are then propagated to flights' next waypoints, and resolutions would happen in chronological order. Meanwhile, the controller intervention count is accumulated. If there was no perturbation in entry times like in deterministic cases, no extra delay and controller intervention should be imposed. In each simulation, the heuristic model is called to check if there is any extra delay and controller intervention. With Monte Carlo simulations, statistics can be calculated so the objectives are evaluated stochastically.

The optimization is implemented using a Non-dominated Sorting Genetic Algorithm (NSGA),  $^{14}$  because of NSGA's ability of handling multiple objective optimization. Because the costs are handled independently, NSGA will lead its search to a Pareto front, where no solution on the front is dominated by another. The dominance with two objectives is defined as: Assuming the objective is to minimize costs and the constraint function g has to be nonnegative, solution A is dominated by solution B only if:

$$\begin{cases} J_{1}(A) > J_{1}(B), & \text{if } g_{A} \ge 0 \text{ and } g_{B} \ge 0, \text{ or, } g_{A} = g_{B} \\ J_{2}(A) > J_{2}(B), & \text{if } g_{A} \ge 0 \text{ and } g_{B} \ge 0, \text{ or, } g_{A} = g_{B} \\ g_{A} < g_{B}, & \text{if } g_{A} < 0 \text{ and } g_{B} < 0, \text{ or, } g_{B} > 0 \text{ and } g_{A} < 0 \end{cases}$$
(5)

where  $J_1$  and  $J_2$  are the objectives and g is the constraint value. In other words, if all three conditions are true, solution A cannot be at the Pareto front.

#### C. Simplified Monte Carlo simulations

Directly incorporating a large scale set of Monte Carlo simulations (like 5,000 simulations) into objective evaluation would be computationally expensive and make the optimization infeasible. Therefore, the number of simulations must be decreased. However, the robustness of stochastic cost evaluation might decrease together with the number of simulations due to the reduced sampling size. For instance, the mean values of extra delay and controller intervention will fluctuate more in the simplified Monte Carlo than in the full-scale Monte Carlo. The inconsistent evaluations make the generated costs inaccurate and may result in incorrect ranking. For instance, solution A is supposed to have a higher rank than solution B because of its low cost, but the opposite situation happened due to the inconsistent evaluations. The more fluctuated the evaluation is, the higher likelihood the ranking is incorrect. Therefore, the number of simulations shouldn't be too low, otherwise, the NSGA's searching process will be misled and good solutions can not be achieved.

In this work the number of simulations was reduced to 1,000, which seemed to be a good compromise between quality and computational time. Figure 2(a) shows the differences in controller intervention between 1,000 and 5,000 simulations during optimization process for around 25,000 solutions. The horizontal axis is the order of each simulation and the vertical axis denotes the differences measured based on the number of controller interventions. The average of intervention errors is 0.19 and the standard deviation is 0.065.

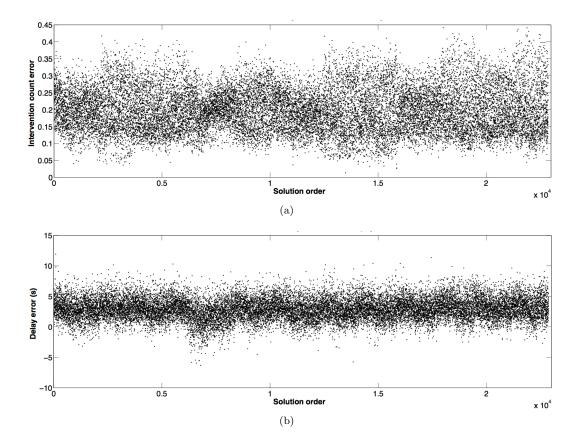


Figure 2. Differences in computing 1,000 simulations vs. 5,000 simulations: (a) Controller intervention count (b) Delay

The controller intervention counts are averaged at 2.0, therefore, the intervention errors between 1,000 and 5,000 simulations are about 9% in terms of average value. Figure 2(b) shows the differences in extra delays between 1,000 and 5,000 simulations. The average of errors is about 2.9 seconds and standard deviation is 1.8 seconds. Given the average delay of 542 seconds, the difference is negligible.

## IV. Results

This section presents the solutions optimized under uncertainty. The trade-offs between delay reduction and controller intervention count increase are shown. The solutions that have similar delays and different intervention counts are compared. Both costs were evaluated stochastically. Monte Carlo simulations were multi-threaded and the optimization took about 6 hours on a MacOS platform with 2x2.66 GHz 6-Core Intel Xeon and 8 GB RAM.

## A. The Pareto front

Given two objectives of reducing delays and controller intervention counts, NSGA led the search to a Pareto front, where no solution on the front was dominated by other solutions. Solutions in final generations of the evolution process were recorded as they were close to the Pareto front. Figure 3 presents these solutions in final generations (1,000+ solutions). Each dot  $(J_1, J_2)$  corresponds to a solution and its coordinates  $J_1$  and  $J_2$  are the two costs of the solution. Both costs were evaluated stochastically using simplified Monte Carlo simulation. The red diamond at coordinates (1227.0, 1.2) sets a reference point for intervention counts and delay costs under spatial separation for which a low intervention count is expected. The costs associated with the reference point were generated as averages from full-scale Monte Carlo simulations of the optimal spatial separation solution, which was produced under deterministic scenarios with delay as the

single objectives. The horizontal red dotted line marks the red diamond's intervention cost which serves as a reference representing low controller intervention. Similarly, the black dotted line at 300 seconds serves as a reference. These dotted lines and diamond solution were used as references for clarity throughout figures in this section.

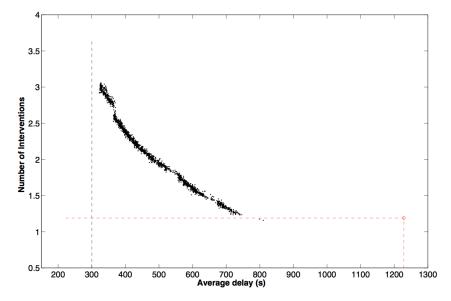


Figure 3. Costs of optimized solutions from a single run where costs were evaluated using simplified Monte Carlo simulations

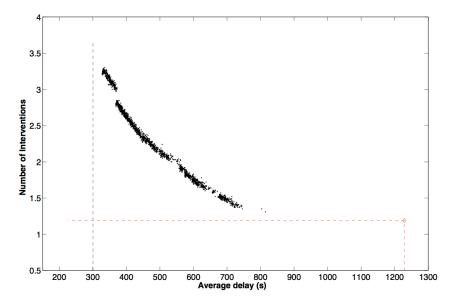


Figure 4. Rectified costs in a post-process where costs were evaluated using a full-scale Monte Carlo simulations

The accuracy of the costs of solutions in Fig. 3 needs to be examined because simplified Monte Carlo simulations were used for evaluation. Therefore, in the post-process, the costs of the same solutions were re-evaluated using full-scale Monte Carlo simulations. The rectified costs are shown in Fig. 4. Using the vertical dotted line as a reference, it can be seen that the solutions in Fig. 3 were shifted up and right in Fig. 4 because of the impact of the robustness imposed on delays and controller intervention. But the marginal difference demonstrated that simplified Monte Carlo simulation works well.

GA-based optimizations are sensitive to initial guesses, which are decided by randomized seeds. Multiple runs are necessary to increase the chance of getting optimal solutions. In this work, ten runs were performed. Figure 5 shows the solutions in final generations for each run. It is noted that different initial guesses led

to different solutions in spite of NSGA's effort, especially when the average delay is low. Figure 6 presents the rectified costs using full-scale simulations in a post-process. As expected, most solutions were shifted up and right. According to the final solutions, it is noted that a clear tradeoff exists between delays and intervention counts. When delay is reduced the chance of controller intervention increases. Variations of intervention counts are large for solutions with similar delays, especially when delays are low. A multiple-objective optimization that incorporates uncertainty can clearly help to find the reduced delay with minimum intervention. If a decision support tool can be built upon this method, in terms of the solutions that are close to the Pareto front, decision makers can choose a "compromise" solution according to their preferences. For instance, if controller intervention is believed to be more important than delay savings, the solutions that have a delay of about 900 seconds and intervention count similar to the spatial solution (red diamond solution) should be picked. Or, if two costs are weighted similarly, the compromise solutions around the middle (e.g. delay around 500) can be chosen because of their large delay savings and tolerable controller intervention increase.

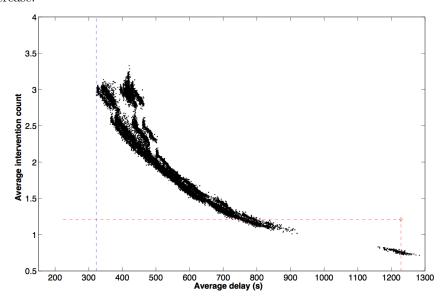


Figure 5. Costs of optimized solutions from ten runs where costs were evaluated using simplified Monte Carlo simulations

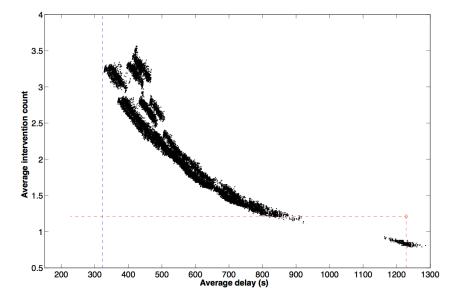


Figure 6. Rectified costs in a post-process where both costs were evaluated using a full-scale Monte Carlo simulation

#### B. Effect of controller intervention

According to Fig. 6, solutions that shared similar delays may cause quiet different controller intervention counts. For instance, at delays around 430 seconds, the average intervention can vary from 2.4 to 3.5. Figures 7 and 8 are the time lines for solutions at either end of this intervention count range. The small gray boxes are the minimum separation requirements associated with flights. As mentioned previously, the separation requirement is a function of aircraft speed and the type of potential conflicts (crossing or in-trail). Therefore, the gray boxes have different lengths. As defined, the boxes are separated by an extra 30 second buffer. Any departure following the direct route should go through WPT2, whereas any arrival from the FIM fix flying the direct route would pass WPT1.

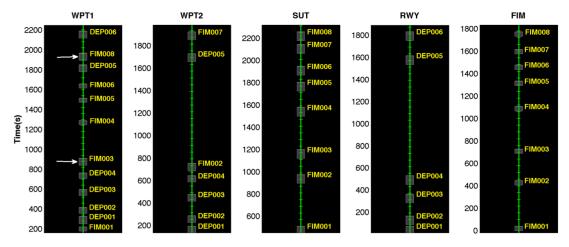


Figure 7. Time line for the solution with high controller intervention count

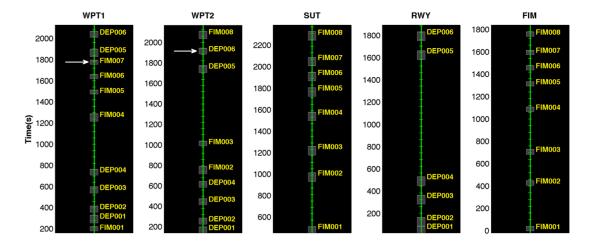


Figure 8. Time line for the solution with low controller intervention count

In terms of costs, these two solutions shared similar delays at around 423 seconds but resulted in different controller intervention efforts. The solution associated with Fig. 8 has an average intervention count of 2.45, whereas the Fig. 7 solution corresponds an average intervention count of 3.56 – a 45 % increase. Based on the comparison, the major differences between them are that: The high-controller-intervention solution allowed FIM003 and FIM008 to fly direct routes whereas the low-controller-intervention solution chose FIM007 and DEP006 to fly direct routes (See the flights pointed by arrows in the figures). Those differences in route options would not affect the delay reduction but would lead to different controller intervention. Apparently, this subtle difference could not be easily predicted without the optimization under uncertainty.

#### C. Discussion

The proposed method presents a set of Pareto solutions to decision makers/air traffic controllers and can help them quickly identify potential benefits and trade-offs for integrated arrival and departure operations. Although this work is focused on LAX, the methodology should be easily applied to different major airports where similar inefficiency exists.

In this work, it took about 6 hours to finish the optimization process, which seemed to be a problem for real-time application. But because both Monte Carlo simulations and the NSGA algorithm are well-suited for parallelization due to their independent calculations and memories, the optimization method proposed in this paper has the potential to be sped up by hundreds of times using GPU implementation <sup>15</sup> and thus may be a candidate for a real-time/fast-time tool. In addition, the searching ranges of decision variables may be smaller in actual operations. For example, the ranges of delays for arrivals are connected to certain traffic control actions like vectoring, holding, and slowing down. Instead of continuous decision variables as in this work, discrete variables that are associated with smaller searching spaces may be used in actual operations. This will help speed up the optimization process as well.

#### V. Conclusions

In order to directly take the impact of uncertainty into account, an optimization method of integrated arrivals and departures under uncertainty was developed in this study. The impact to controller intervention was included in the optimization. The problem was formulated as a multiple objective optimization with delays and intervention count as the costs. Monte Carlo simulations were utilized to stochastically evaluate both costs. To enable the time-consuming optimization, the number of Monte Carlo simulations were reduced. The Pareto fronts that contain sets of solutions were identified. The trade-off between delays and controller intervention counts was shown and solutions with similar delays but different intervention effort were investigated.

Through this study, the optimization under uncertainty for integrated arrivals and departures were found to be feasible with simplified Monte Carlo simulations. Decreasing the number of simulations from 5,000 to 1,000 affected the controller intervention evaluation 9% but reduced the computational times to a reasonable level. Using this formulation, the method can provide a sweep of solutions that are close to the Pareto front so the decision makers can choose in terms of their preferences. Solutions may have similar delays but cause quite different controller intervention efforts. The subtle difference in solutions/delays can result in a significant difference in intervention counts (e.g. 45%), which would not be easily foreseen without the optimization under uncertainty. As the Monte Carlo simulations and the NSGA algorithm are all well-suited for parallelization because of their independent calculations and memories, the proposed optimization algorithm is expected to be sped up by hundreds of times using GPU implementation and becomes a real-time/fast-time application.

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